ALGEBRA

C – 6

49. If $x^4 - 62x^2 + 1 = 0$, where x >0, *then the* value of $x^3 + \frac{1}{x^3}$ is:

(A) 488 (B) 364
(C) 512 (D) 500

SSC CGL 2021

$$x^2 - 62 + \frac{1}{x^2} = 0$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 62$$

$$x + \frac{1}{x} = \sqrt{62+2} = \sqrt{64} = 8$$

$$x^3 + \frac{1}{x^3} = (8)^3 - 3(8) = 512 - 24 = 488$$

50. If $a+\frac{1}{a}=2$, then $a^4-\frac{1}{a^4}$ is equal to:

(A) 0

(B) $\frac{1}{4}$

(C) 1

(D) 4

SSC CHSL 2019

$a = 1$

51. If $\left(x^2+\frac{1}{x^2}\right) = 6$ then find the value of $\left(10x - \frac{10}{x}\right)$

(A) +/-15
(B) +/-20
(C) +/-30
(D) +/-40

RRB NTPC 2016

$$x^2+\frac{1}{x^2}=6$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2+2\cdot x\cdot\frac{1}{x}=6$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=4$$

$$\Rightarrow x-\frac{1}{x}=\pm 2$$

$$10x-\frac{10}{x}$$

$$=10\left(x-\frac{1}{x}\right)$$

$$=\pm 20$$

51. If $\left(x^2+\frac{1}{x^2}\right) = 6$ then find the value of $\left(10x - \frac{10}{x}\right)$

(A) +/-15
(B) +/-20
(C) +/-30
(D) +/-40

RRB NTPC 2016

$x^2+\frac{1}{x^2}=6$

$\Rightarrow x+\frac{1}{x}=\sqrt{6+2}$
$=\sqrt{8}$
$=2\sqrt{2}$

$\Rightarrow x-\frac{1}{x}=\sqrt{(2\sqrt{2})^2-4}$
$=\sqrt{4}=2$

$x^2+\frac{1}{x^2}=6$

$x-\frac{1}{x}=\sqrt{6-2}$
$=\sqrt{4}=2$

52. If $X + \frac{1}{X} = 2$ then find the value of $(X^4 + \frac{4}{X^4})(X^3 - \frac{4}{X^3})$

$(1 + \frac{4}{1})(1 - \frac{4}{1})$

(A) -15 (B) 15

(C) -10 (D) 10

SSC CPO 2024

$x = 1$

$5 \times (-3)$

$= -15$

53. If $16X^4 + \frac{1}{16x^4} = 14159$ then find the value of $2X + \frac{1}{2X}$.

(A) 9 (B) 11
(C) 12 (D) 15

SSC CPO 2024

$4x^2 + \frac{1}{4x^2} = \sqrt{14159+2}$

$= \sqrt{14161}$

$= 119$

$2x + \frac{1}{2x} = \sqrt{119+2}$

$= \sqrt{121}$

$= 11$ A

54. If $X\left(5 - \frac{2}{X}\right) = \frac{5}{X}$ then the value of $X^2 + \frac{1}{X^2}$ is:

(A) $\frac{54}{23}$

(B) $\frac{53}{28}$

(C) $\frac{53}{27}$

(D) $\frac{54}{25}$ ✓

SSC CPO 2023

$$x\left(5 - \frac{2}{x}\right) = \frac{5}{x}$$

$$\Rightarrow 5x - 2 = \frac{5}{x}$$

$$\Rightarrow 5x - \frac{5}{x} = 2$$

$$\Rightarrow 5\left(x - \frac{1}{x}\right) = 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \frac{2}{5}$$

$$x - \frac{1}{x} = \frac{2}{5}$$

$$x^2 + \frac{1}{x^2} = \left(\frac{2}{5}\right)^2 + 2$$

$$= \frac{4}{25} + 2 = \frac{54}{25}$$

55. If $2X + \frac{2}{X} = 5$ then the value of $(X^3 + \frac{1}{X^3} + 2)$ is:

(A) $\frac{81}{11}$

(B) $\frac{81}{7}$

(C) $\frac{71}{8}$

(D) $\frac{81}{8}$

SSC CHSL 2023

$$2x + \frac{2}{x} = 5$$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \frac{5}{2}$$

$$\therefore x^3 + \frac{1}{x^3} = \left(\frac{5}{2}\right)^3 - 3\left(\frac{5}{2}\right)$$

$$= \frac{125}{8} - \frac{15}{2}$$

$$= \frac{125}{8} - \frac{60}{8} = \frac{65}{8}$$

$$\frac{65}{8} + 2$$

$$= \frac{81}{8}$$

$$\begin{array}{r} 65 \\ 16 \\ \hline 1 \end{array}$$

56. If $(2X - \frac{3}{X}) = 2$, then what is the value of $(16X^4 + \frac{81}{X^4})$?

(A) 184 ✓
(B) 328
(C) 180
(D) 220

SSC CGL 2020

$$\left(2x - \frac{3}{x}\right)^2 = (2)^2$$

$$\Rightarrow 4x^2 - 2 \cdot 2x \cdot \frac{3}{x} + \frac{9}{x^2} = 4$$

$$\Rightarrow \left(4x^2 + \frac{9}{x^2}\right)^2 = (16)^2$$

$$\Rightarrow 16x^4 + \frac{81}{x^4} + 2 \cdot 4x^2 \cdot \frac{9}{x^2} = 256$$

$$= 16x^4 + \frac{81}{x^4} = 256 - 72 = 184$$

Yourstudy

$x - \frac{1}{x}$

$$x + \frac{1}{x} = 4$$

$$\left(x - \frac{1}{x}\right)^2$$

$$= \left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}$$

$$= \sqrt{(4)^2 - 4 \times x \times \frac{1}{x}}$$

$$x + \frac{1}{2x} = 4$$

$$\left(x - \frac{1}{2x}\right)^2$$

$$= \left(x + \frac{1}{2x}\right)^2 - 4 \cdot x \cdot \frac{1}{2x}$$

$$= \sqrt{(4)^2 - 4^2 \times x \times \frac{1}{2x}}$$

$$= \sqrt{16 - 2} = \sqrt{14}$$

64. If $(5a + \frac{4}{a} - 2) = 13$ and $a > 0$ what is the value of $(25a^2 + \frac{16}{a^2})$?

(A) 158
(B) 157
(C) 185
(D) 175

SSC CHSL 2023

$5a + \frac{4}{a} = 15$

$(5a)^2 + \left(\frac{4}{a}\right)^2$

$= (15)^2 - 2 \times 5a \times \frac{4}{a}$

$225 - 40$

$= 185$